THE MISPRICING OF CALLABLE U.S. TREASURY BONDS: A CLOSER LOOK

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INTRODUCTION

An ongoing controversy exists concerning the valuation of callable U.S. Treasury bonds. Recent studies by Longstaff (1992) and Edleson, Fehr, and Mason (1993) compare market prices for callable and noncallable Treasury issues in an attempt to estimate the value of the options embedded in the callable bond. Surprisingly, both studies find that negative option values are very common, implying that callable Treasury bonds are significantly overpriced. In contrast, in an expanded analysis, Jordan, Jordan, and Jorgensen (1995) find that negative option values very rarely occur.

Most recently, Carayannopoulos (1995) reexamines the issue with a broader sample of callable bonds and, as in Longstaff and in Edleson et al., reports that negative option values are quite frequent, thereby reopening the debate. Furthermore, unlike previous studies, Carayannopoulos (1995) reports that negative implied put values are much more common than negative implied call values. Carayannopoulos also documents a coupon effect indicating that negative option values are concentrated in lower coupon callable bonds.

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With the use of the same approach as Carayannopoulos (1995), this article provides the most extensive analysis of this issue to date. Based on an examination of almost 300,000 implied option values over the period 1990–1994, negative option values large enough to overcome the bid–ask spread are observed 8.9% of the time. This is somewhat larger than the percentage reported in Jordan et al. (1995), but it is substantially smaller than that reported in other studies. Further, contrary to Carayannopoulos, the results show that negative implied put values occur much less frequently than negative implied call values and that Carayannopoulos's conclusion to the contrary is based on an erroneous classification of puts and calls.

A detailed analysis indicates that the coupon effect in Carayannopoulos is related to premium/discount status rather than coupon rates *per se*. In addition, the approach used to value implied options, which relies completely on U.S. Treasury STRIPS prices, tends to undervalue implied options. Negative option values are observed much more frequently when the true value is small and the undervaluation problem is most severe. Overall, the empirical results provide support for theoretical arguments in Jordan et al. (1995) regarding problems that arise when STRIPS prices are used to value callable Treasury bonds.

The remainder of the study is organized as follows. The next two sections discuss valuation of the implicit options in callable bonds and review previous research in this area. An evaluation of the estimation procedure used in this article is then presented, followed by empirical results on the value of implied options and a discussion of the apparent mispricing of callable Treasury bonds. A summary concludes the article.

BACKGROUND

The call feature on a Treasury issue is relatively simple. The bonds are callable at par and call protected outside the last 5 years of their lives. Thereafter, the bonds are callable on their semi-annual coupon dates, so there are 10 call dates before maturity. The call provision is thus a call option with an exercise price equal to par and a maturity equal to that of the bond. It is a European option outside the last five years of its life; it converts to a semi-American or Bermudan option when the call protection expires.[1]

Because the call feature gives the Treasury the right to redeem the bond before maturity, a callable Treasury bond must sell for less than a

[1]The term *semi-American* indicates that the option can be exercised only on certain dates before maturity; *Bermudan* options are those that are "between" American and European options.

noncallable Treasury bond with the same coupon and maturity. The difference between their prices is the implicit value of the embedded call option. If the callable bond sold for more than an otherwise identical noncallable issue, the implication would be that the right to call the bond has a negative value, or, equivalently, that the callable bond is overvalued.

Additional bounds can be placed on the price of a callable Treasury bond. Assuming the Treasury follows a rational call policy, a callable Treasury bond can sell for at most par value on a call date.[2] Given this, as pointed out in Jordan et al. (1995), a callable Treasury bond should not sell for more than a noncallable issue with the same coupon that matures on any call date. For example, the difference in the value of a callable issue and a noncallable bond that matures five years earlier can be viewed as the implied value of a European put on the callable bond. This put option, in effect, gives the government the right to force the holder of a *t*-5-year noncallable bond to accept a 5-year callable bond at time *t*-5 in lieu of par, so its value should never be negative.

Valuing the implicit option on a callable Treasury bond requires estimating the value of a noncallable bond with the same coupon as the callable bond and a maturity corresponding to one of the call dates or final maturity. Three similar methods have been used to estimate the value of these noncallable bonds: the striplets approach, the triplets approach, and the all-STRIPS approach. The striplets approach uses a U.S. Treasury coupon STRIPS and a coupon bond to synthesize a noncallable bond with the desired coupon, whereas the triplets approach uses two coupon-bearing Treasuries. The final approach, the all-STRIPS approach, uses a portfolio made up strictly of coupon STRIPS to replicate all of the cash flows on the noncallable bond.

The three approaches have distinct advantages and disadvantages. With the triplets approach, only coupon bonds are used, thereby making use of the most comparable instruments, but, because of data availability, only a relatively limited number of callable bonds can be examined. The striplets method allows a somewhat larger number of callable bonds to be examined. However, as discussed in Jordan et al. (1995), STRIPS are taxed disadvantageously, particularly relative to discount coupon Treas-

[2]The optimal call policy is to call a bond if the price on a call date would otherwise exceed par value. Longstaff (1992) reviews Treasury calls and concludes that the Treasury has historically followed a near-optimal policy. Because Treasury bonds must be called 4 months before a coupon date, the actual optimal strategy is to call if the bond's price would otherwise exceed the price of a 4-month Treasury bill on a notification date. Edleson et al. (1993) consider the notification period and conclude that the Treasury's policy has been closer to optimal than indicated by Longstaff. Bliss and Ronn (1995) provide a detailed review of Treasury calls and the optimal call strategy. Since Longstaff's study, several more issues have been called, most recently (in April 1996) the 8% August 1996-01 issue. In each case, the call was optimal or near-optimal.

uries. The striplets approach may therefore undervalue the noncallable issue, which, in turn, can create the appearance of negative option values. Finally, the all-STRIPS approach allows the analysis of a much larger set of bonds, but the STRIPS tax disadvantage is potentially more severe. In addition, up to 40 separate STRIPS must be combined to synthesize the needed noncallable bond, thereby potentially producing much noisier estimates.

PREVIOUS RESEARCH

Longstaff (1992) uses the striplets approach to investigate implied call option values and finds that 61.5% of the call values are negative when estimates are based on the midpoint of the bid and ask prices, whereas 50.7% of the negative call estimates (or 31.2% of all call estimates) are large enough to generate profits even after considering the bid-ask spread. Edleson et al. (1993) extend Longstaff's study by examining both the final maturity date (the implied call) and the first call date (the implied put). With the use of the striplets approach, they find that 56.5% of the estimated call option values and 20.2% of the put values are negative after accounting for the spread.

Jordan et al. (1995) use the triplets approach to value implied options. They extend earlier studies by examining each of the intermediate call dates, as well as the implied call option and the implied put option. Their estimates of implied option values are overwhelmingly positive; option values negative enough to overcome the bid–ask spread only occur about 3% of the time. They reconcile their results with the anomalous findings in previous studies by showing that the methods used in those studies can lead to the appearance of a negative option value when the true value is positive.

Carayannopoulos (1995) uses the all-STRIPS approach to value the implied put and the implied call for 23 callable bonds over the period July 1989 to June 1993. Based on 1,078 monthly, midmarket prices, he finds 65 negative call values and 282 negative put values. On a spread-adjusted basis, 188 (17.4%) of the put values are still negative, as are 28 (2.6%) of the call values. Thus, in contrast to previous studies, Carayannopoulos reports that negative implied put option values are observed frequently, whereas negative call options are only rarely observed.

In examining the price behavior of individual bonds, Carayannopoulos finds that for bonds with a coupon rate of 10% or higher, negative option values are infrequent. In contrast, for those with a coupon rate of 8.75% or lower, violations are quite common. Thus, he documents a systematic difference in the pricing of high and low coupon callable bonds and concludes that the negative option values in earlier studies are tied to a coupon effect.

In comparing the results in Carayannopoulos to those in previous studies, it is important to recognize a difference in reporting conventions. Previous studies report the percentage of observed option values that are negative. Carayannopoulos, however, frequently focuses on the percentage of callable bond prices that imply a negative option value. The difference can be illustrated with a simple example.

A hypothetical sample consists of 100 callable bond observations and, therefore, 100 implied calls and 100 implied puts. Of these, 20 calls and 10 puts (with no overlap) have negative values. Previous studies would report that, of 200 implied option values, 30, or 15%, are negative. Carayannopoulos, on the other hand, would report that, of 100 callable bond observations, 30, or 30%, contain a negative option value. Were it not for overlaps (the same observation implying both a negative put and a negative call), the percentages in Carayannopoulos would simply be twice as large. Neither approach is incorrect, but the net effect is that care must be taken in comparing relative frequencies across studies.

DATA AND EVALUATION OF THE ALL-STRIPS APPROACH

This section investigates the ability of the all-STRIPS approach to price noncallable bonds. This is an important issue because any errors in valuing the noncallable bond underlying a callable bond are translated one-for-one into misvaluation of implied options. Further, theoretical considerations in Jordan et al. (1995) lead them to specifically conclude that STRIPS-based approaches may systematically undervalue noncallable bonds and, therefore, implied options, but this conclusion is not investigated empirically. The undervaluation is predicted to be more pronounced for implied call options.[3]

[3]A direct comparison of the all-STRIPS and triplets approaches for valuing implied options is only possible for six callable bonds. The two approaches can be used to value the implied call option for one of these bonds and the implied put value for the remaining five. For the implied put value, the triplets approach results in negative values 4.4% of the time, whereas the all-STRIPS approach produces negative values 3.4% of the time. The mean of the difference in the implied values from the two methods is 2¢. The results for the implied call are quite different. The triplets approach generates negative implied values for only 3.2% of the observations, whereas the all-STRIPS approach results in negative implied values for 19.7% of the sample. The mean difference in the implied call values is 30¢, which is both statistically (the *t* statistic is 23.12) and economically significant. Given the limited number of bonds available for comparison of the two methods, these results should be interpreted with caution; however, they are fully consistent with the theoretical predictions of Jordan et al. (1995).

TABLE I

Results from Pricing Noncallable Notes and Bonds with Portfolios of Coupon STRIPS

Sample	*No. of Observations*	*% Pos*	*Avg(ΔP)*	*Std(ΔP)*	*Max*	*0.90*	*0.75*	*Med*	*0.25*	*0.10*
				A. All Noncallable Issues						
Full	132,641	61%	$0.11	$0.32	$6.01	$0.46	$0.21	$0.04	($0.05)	($0.13)
Premium	117,363	58%	$0.09	$0.30	$6.01	$0.44	$0.19	$0.03	($0.05)	($0.14)
Discount	15,278	78%	$0.19	$0.44	$2.21	$0.57	$0.33	$0.15	$0.02	($0.06)
				B. All Noncallable Strippable Issues						
Full	63,084	71%	$0.17	$0.31	$6.01	$0.55	$0.28	$0.11	($0.02)	($0.11)
Premium	54,859	68%	$0.15	$0.29	$6.01	$0.50	$0.24	$0.08	($0.03)	($0.12)
Discount	8,225	94%	$0.35	$0.32	$2.21	$0.77	$0.48	$0.28	$0.14	$0.04

Summary statistics for pricing deviations from noncallable (Panel A) and noncallable, strippable (Panel B) Treasury issues priced with a portfolio of coupon STRIPS. Daily prices cover the period 1990–1994 for all noncallable issues. All values are relative to a $100 face value. The price deviation, ΔP, is calculated as the issue bid price less the STRIPS portfolio bid price. A positive value indicates that the STRIPS portfolio undervalues the cash market note or bond. *Discount* and *Premium* indicate that the sample is separated into discount and premium issues.

The STRIPS data set used in this study consists of nearly 150,000 daily bid and ask yield quotations on coupon Treasury STRIPS obtained from Street Software Technology, Inc., covering the period January 1990 through December 1994. The underlying data source is the STRIPS desk of Bear Stearns, Inc., one of the largest Treasury dealers in the world; the same data are used for the quotations supplied daily to *The Wall Street Journal*. For each day over the 5-year period covered by the STRIPS data, bid price quotations on all outstanding Treasury issues are obtained from the Federal Reserve Bank of New York. As with the STRIPS data, these are the prices reported daily in *The Wall Street Journal*.

Rather than using STRIPS prices as they appear in *The Wall Street Journal*, prices are instead calculated from the supplied yields with the use of standard market practice. Quotations in the STRIPS market are in terms of yield; prices are computed from these yields and rounded to the nearest cent per $1,000,000 face value. However, prices reported in *The Wall Street Journal* are rounded to the nearest $\frac{1}{32}$ per $100 face value; hence, they include an artificial source of noise.

In addition, prices in *The Wall Street Journal* are computed from the supplied yields based on skip-day settlement procedures, whereas standard practice in the Treasury market (including STRIPS) is for next-day settlement. Hence, daily STRIPS prices are computed from the dealer bid and ask yield quotations with the use of standard market practice, and these prices are used in all the estimations in this article. Similarly, full prices (price plus accrued interest) on all bonds are calculated assuming next-day settlement. Consequently, STRIPS prices and bond prices are synchronized in terms of settlement procedures, thereby reducing the noise from this source.

With the use of the daily data, a suitably constructed portfolio of coupon STRIPS is used to value every noncallable Treasury issue making coupon payments on the same quarterly cycle as Treasury STRIPS, for a total of 132,641 comparisons.[4] It appears that no such extensive comparison of coupon-bearing issues versus STRIPS-synthesized issues currently exists in the literature. Panel A of Table I provides summary statistics on the pricing deviations for these issues. The price deviation, ΔP, is calculated as the coupon bond bid price less the STRIPS portfolio bid price.[5] Thus, a positive value indicates that the STRIPS portfolio undervalues the cash market bond.

As shown in Panel A of Table I, the STRIPS portfolio undervalues the cash market bond in 61% of the observations; this frequency exceeds 50% at any significance level based on a standard proportions test. The average misvaluation is $0.11 per $100 face value, which, in light of the sample size, is similarly significant. Examining the various percentile results, the median misvaluation is positive, but less than the mean, and 50% of the observed misvaluations lie between $0.21 and −$.05.

Panel A of Table I also reports results after dividing the sample into premium and discount observations.[6] The results show that the tendency for the STRIPS portfolios to undervalue coupon bonds is much more pronounced for discount bonds. The STRIPS-estimated price is too low 78% of the time, and the average misvaluation is $0.19. The average misvaluation is thus about twice as large for discount issues. Substantial misvaluations occur on occasion; the maximum observed difference is $6.01 for the premium bonds and $2.21 for the discount bonds.[7,8]

[4]Only issues with original maturities of 10 or more years are eligible for stripping. Prior to 1996, such issues were only auctioned on the Treasury's quarterly refunding cycle. Consequently, the STRIPS used in this article mature on the 15th of February, May, August, and November. As in other studies, only coupon STRIPS are used because of their generic nature relative to principal STRIPS, which are issue specific.

[5]Bid prices are used rather than midmarket prices because the daily coupon bond dataset contains only bid quotes. The ask prices in the New York Fed data (the prices reported in *The Wall Street Journal* and supplied by CRSP) are generated by the Fed based on dealer-supplied bid quotes and are not necessarily representative of actual ask prices (Duffee, 1996).

[6]Throughout this article, issues are classified as premium or discount based on their quoted prices (prices net of accrued interest). This is consistent with industry practice and is the correct classification for tax purposes.

[7]Newly issued, on-the-run Treasuries often carry a price premium. Nevertheless, when on-the-run

Carayannopoulos (1995) performs an analysis similar to that in Table I, but he reaches the opposite conclusion, finding that STRIPS portfolios tend to overvalue coupon bonds by 25¢ on average. He also indicates that there is no significant difference between high and low coupon bonds. These conclusions are based on an analysis of 30-year bonds eligible for stripping. However, this may not be the best comparison group, because, with the exception of the November 2009-14 issue, callable bonds cannot be stripped. In addition, because the Treasury stopped issuing callable bonds when the STRIPS program began in 1985, every noncallable 30-year bond eligible for stripping has a maturity greater than that of every callable bond.

Nonetheless, by way of comparison, Panel B of Table I examines the subset of issues in Panel A that are eligible for stripping, including notes and bonds. As shown, contrary to results in Carayannopoulos, the undervaluation documented for the entire sample is not only present in this subset of strippable issues, it is even more pronounced. The average discount issue misvaluation is over two times larger than the premium issue misvaluation for this set. When the analysis in Panel B is repeated for only 30-year strippable bonds, the results are essentially the same and thus are not reported separately.

Overall, the results in Table I show that coupon STRIPS portfolios undervalue noncallable coupon issues by a small amount on average. The underpricing is more pronounced for discount issues than for premium issues. Further, estimates can be noisy and extreme pricing errors occur on occasion. One implication of these findings is that in circumstances where the true option value is small but positive, apparent negative option values will likely be found. Although explanations for the tendency of STRIPS portfolios to undervalue coupon-bearing issues are not formally examined in this article, these results are fully consistent with the tax disadvantage argument in Jordan et al. (1995). The fact that the underpricing is more pronounced for discount bonds is especially suggestive in this regard, because the STRIPS tax disadvantage is more substantial relative to discount bonds.

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observations are excluded, very similar results are obtained. The average price deviation is $0.10 for the full sample, $0.19 for discount observations, and $0.09 for premium observations.

[8]To investigate whether spreads are an important determinant of these findings, the analysis in Table I is repeated with the use of midmarket prices. The results are qualitatively the same. In addition, two subsamples are examined: those with spreads of $\frac{2}{32}$ (65.6% of the sample) and those with spreads of $\frac{4}{32}$ (29.2% of the sample). As with the full sample, the degree of underpricing is greater for discount issues than it is for premium issues in both of the subsamples.

EMPIRICAL RESULTS

This section presents estimates of the option values embedded in callable Treasury bonds. The all-STRIPS approach is used to examine all 22 callable bonds with final maturities of 1998 or beyond. This is the same set examined in Carayannopoulos (1995) except for the 7.5% August 1988–93 bond.[9]

The implied option value on day t is estimated as

$$Option_t = \sum_{j=1}^{N} [(c/2)S_{j,t}] + S_{N,t} - P_t \qquad (1)$$

where c is the annual coupon rate of the callable bond, S denotes the current price of the coupon STRIPS with maturity j, and P denotes the full (with interest) price for the cash bond. Let M represent the number of payment dates until maturity of the callable bond. Then the implied call option is estimated with $N = M$, the implied put option is estimated with $N = M - 10$, and the intermediate date option values are estimated with $N = M - k$, where $k = 1, 2, \ldots, 9$.

The basic results are summarized in Table II. Part A of the table considers the full sample; Parts B and C focus on premium and discount bonds, respectively. As in Jordan et al. (1995), all 11 call dates are evaluated for each bond first, and then the implied put (the first call date, $N = M - 10$) and implied call (final maturity date, $N = M$) are examined separately.

Beginning with the full sample and all 11 call dates, Part A of Table II shows that there are a total of 293,832 implied option values. On a spread-adjusted basis, 8.9%, or about 26,000, are negative. This is larger than the 2.3% reported in Jordan et al., but still much smaller than that reported in Longstaff (1992) and Edleson et al. (1993). With the full sample, there are 26,712 implied calls and the same number of implied puts. On a spread-adjusted basis, 18.9% of the implied calls have a negative value compared to only 4.0% of the implied put options.[10]

Segregating the sample into premium and discount bonds reveals an interesting result. For the 256,355 premium observations, negative option

[9]This bond is not included for three reasons. First, because the bond was callable as of August 1988, and the STRIPS data begin in January 1990, there are no observations for the implied put value. Second, this issue is the only one for which the call protection had expired as of the beginning of the sample period. Finally, the bond was called in October 1991, so only a relatively limited amount of data is available for it. Nevertheless, for the sake of completeness, all of the analyses in this article are repeated with this bond included; the results are not affected.

[10]If the analysis in Table II is repeated with the use of midmarket prices obtained from the monthly CRSP Government Bond file, the percentage of observations with negative implied options values lies between the percentages reported in Table II for bid–bid prices and spread-adjusted prices.

TABLE II

Estimated Option Values with the Use of the All-STRIPS Approach

				% Negative								
Sample	*Implicit Option*	*Obs.*	*Avg. Value*	*Bid Prices*	*Sprd. Adjst.*	*Std*	*Max*	*0.90*	*0.75*	*Med*	*0.25*	*0.10*
A. Full	All	293,832	$2.18	12.4	8.9	$2.18	$15.26	$5.12	$3.25	$1.76	$0.66	($0.15)
	Call	26,712	$3.28	21.3	18.9	$3.24	$15.26	$7.46	$5.45	$3.13	$0.38	($0.68)
	Put	26,712	$0.82	9.0	4.0	$0.90	$7.67	$1.76	$1.07	$0.67	$0.34	$0.03
B. Premium	All	256,355	$2.44	7.2	4.9	$2.16	$15.26	$5.37	$3.49	$1.98	$0.96	$0.22
	Call	23,305	$3.87	10.4	8.4	$3.05	$15.26	$7.73	$5.83	$3.72	$1.71	($0.05)
	Put	23,305	$0.70	8.7	3.9	$0.61	$3.05	$1.52	$0.99	$0.65	$0.33	$0.04
C. Discount	All	37,477	$0.39	47.7	35.9	$1.40	$7.67	$2.16	$0.85	0.05	($0.43)	($0.96)
	Call	3,407	($0.75)	96.1	90.5	$0.54	$1.08	($0.21)	($0.38)	($0.61)	($1.08)	($1.57)
	Put	3,407	$1.62	10.8	4.4	$1.76	$7.67	$4.79	$2.31	$1.05	$0.39	($0.02)

Estimates of implied option values with the use of the all-STRIPS approach presented in eq. (1). Option values are derived from daily bid prices on 22 callable Treasury bonds during the period 1990–1994. All values are relative to a face value of $100. In Parts B and C, the sample is segregated into premium and discount issues. *All* refers to all 11 call dates; *Call* is the implied call option value; *Put* is the implied put option value.

values are observed 4.9% of the time when all 11 call dates are considered. For the 23,305 implied call options, 8.4% are negative; the corresponding percentage for implied puts is 3.9%.

However, when one examines the discount observations, negative option values appear to occur quite frequently. On a spread-adjusted basis, negative values are observed 35.9% of the time when all call dates are considered. Even more striking is the fact that of 3,407 implied call values, 90.5% are negative on a spread-adjusted basis, whereas only 4.4% of the implied put values are negative.

The results in Table II clearly lead to two conclusions. First, apparent negative option values are much more frequent for discount bonds. Second, when the analysis is restricted to just implied puts and calls, there are almost five times as many negative calls as negative puts on a spread-adjusted basis. This second conclusion is surprising for two reasons.

First, approximately 87% of the observations consist of premium bonds. For callable bonds selling at a premium, the call option is in the money and the put option is out of the money.[11] Consistent with this, in the premium sample, the average put value is only \$0.70, compared to \$3.87 for the implied call. Given the noise in the estimation procedure described earlier and the generally small put values, the fact that only 3.9% of the put values are negative for these premium issues strongly suggests that economically significant negative put values do not occur.

Second, the results on the relative frequency of negative puts versus negative calls are the opposite of what Carayannopoulos (1995) reports. In an attempt to reconcile these conflicting findings, the data underlying Table II are used to replicate the analysis in Carayannopoulos. The subsample considered does not perfectly match Carayannopoulos's sample, however, because his data begin 6 months earlier (and end 18 months earlier) and include the 7.5% August 1988–1993 bond. Nonetheless, by examining the last day of each month and truncating the data to end at the same time as Carayannopoulos's, the resulting sample contains 918, or about 85%, of his 1,078 observations. Table III, which is constructed to match Table I in Carayannopoulos, reports the results.

In Table III, Panel A compares bid prices (versus midmarket prices in Carayannopoulos), and Panel B examines spread-adjusted violations.

[11]The underlying asset for the implied call option is a noncallable bond with maturity and coupon equal to that of the callable bond. The call option is thus in the money when the noncallable bond's price exceeds par. However, if the callable bond's price is below par, the option could still be in the money. For the implied put option, the underlying asset is the callable bond, so its moneyness can be directly observed. Also, in both cases, the relevant price for determining moneyness is the quoted price (the price net of accrued interest) relative to par (ignoring the small error from linear accrual on the coupon).

TABLE III

Boundary Violations and Negative Option Values from Month-End Data

	Number of Violations	*Mean*	*Std*	*Max*	*0.90*	*0.75*	*Median*	*0.25*	*0.10*
	Panel A: Replication with the Use of Bid Prices (918 Observations)								
Boundary	301	\$0.73	\$0.58	\$2.78	\$1.56	1.08	\$0.58	\$0.29	\$0.12
Short	72	\$0.25	\$0.25	\$1.06	\$0.60	\$0.35	\$0.15	\$0.06	\$0.02
Long	269	\$0.79	\$0.58	\$2.78	\$1.65	\$1.23	\$0.62	\$0.33	\$0.18
	Panel B: Replication With Bid–Ask Spread Considered (918 Observations)								
Boundary	247	\$0.64	\$0.54	\$2.53	\$1.42	\$1.03	\$0.46	\$0.21	\$0.07
Short	26	\$0.26	\$0.20	\$0.87	\$0.48	\$0.37	\$0.24	\$0.11	\$0.02
Long	238	\$0.65	\$0.54	\$2.53	\$1.42	\$1.05	\$0.48	\$0.22	\$0.08

Reconstruction of Table I in Carayannapoulos (1995). *Boundary* refers to the number of observations implying a negative put, call, or both. The summary statistics for boundary are based on the more negative value if both the implied put and call are negative for a particular observation. *Short* refers to the short maturity synthetic noncallable bond and reports the number of negative put values. This corresponds to $N = M - 10$ in eq. (1). *Long* refers to the long-maturity synthetic bond and reports the number of negative call values, which corresponds to $N = M$ in eq. (1). All values are relative to a \$100 face value. The values in the table are calculated with the use of only those observation for which a violation of the indicated type occurs.

Following Carayannopoulos, the label *boundary* in the table refers to the number of observations that contain either a negative call or a negative put (or both). The label *long* refers to the long-maturity synthetic bond and indicates the number of implied negative call values. Similarly, the term *short* refers to the short-maturity bond and indicates the number of negative put values. The means (and other values) in the table are calculated with only those observations for which a violation of the indicated type occurs.

A comparison of Table III here to Table I in Carayannopoulos shows that the overall frequency with which negative option values occur is similar. However, it appears that Carayannopoulos inadvertently reverses implied puts and calls. As a result, he substantially understates the frequency of negative calls and similarly overstates the frequency of negative puts. More seriously, this misclassification leads Carayannopoulos to his central conclusion that an unusual, and previously undocumented, coupon effect exists in callable bond prices in which bonds with coupon rates of 8.75% or less imply negative call *and* put values.

In contrast, Table I in this study suggests that the appearance of negative option values is related to premium/discount status. Of course, because these bonds are observed over the same time period, the ones with lower coupons necessarily have lower prices, so these two effects cannot be completely separated. Table IV examines this issue in more detail by dividing the data into five categories based on price. The specific

TABLE IV

Effect of Premium/Discount Status

	All P	*P < $90*	*90 ≤ P < 98*	*Approx Par*	*102 < P ≤ 110*	*P > 110*
Total observations	26,712	172	2,259	2,797	5,681	15,803
Panel A: Bid Prices						
Negative calls	5,695	172	2,185	1,967	1,289	82
	(21.3%)	(100.0%)	(96.7%)	(70.3%)	(22.7%)	(0.5%)
Negative puts	2,406	0	123	676	494	1,113
	(9.0%)	(0.0%)	(5.4%)	(24.2%)	(8.7%)	(7.0%)
Panel B: Spread-Adjusted Prices						
Negative calls	5,050	170	2,065	1,781	982	52
	(18.9%)	(98.8%)	(91.4%)	(63.7%)	(17.3%)	(0.3%)
Negative puts	1,061	0	25	350	266	420
	(4.0%)	(0.0%)	(1.1%)	(12.5%)	(4.7%)	(2.7%)

Frequency of negative puts and calls in five bid-price ranges. Put and call values obtained with the use of the all-STRIPS approach presented in eq. (1) from daily prices on 22 callable Treasury bonds covering the period 1990–1994. Numbers in parentheses are the percentage of observations in the indicated price range with negative implied option values.

cutoffs are arbitrarily chosen, but they are simply intended to separate the prices into deep discount ($P < 90$), discount ($90 \leq P < 98$), near-par ($98 \leq P \leq 102$), premium ($102 < P \leq 110$), and super premium ($P > 110$) classes.

Examining Panel A of Table IV, which is based on bid prices, only 172 deep discount observations exist in the sample; however, 100% of them imply negative call option values. For the discount case, there are 2,259 observations, of which 96.7% imply negative call values. For the super premium group (which contains approximately 60% of the sample), negative call values occur less than 1% of the time. At the same time, negative put values are almost never observed in the two discount categories. On a percentage basis, negative put values are most frequent near par, and then they become somewhat less common. However, it should be remembered that relatively few negative put values exist in the first place.

The results in Table IV lead to several basic conclusions. First, for observations with prices above 110, negative call values are practically nonexistent, regardless of the coupon rate on the underlying issue. Second, negative put values are relatively uncommon, particularly on a spread-adjusted basis, and premium/discount status does not appear to be a major determinant. Finally, negative call option values appear to occur quite frequently for issues selling for par or below.

To determine if the results in Table IV are associated with the coupon effect described in Carayannopoulos, Table V repeats the analysis for

TABLE V

Effect of Coupon Rate and Premium/Discount Status

	All P	*P < $90*	*90 ≤ P < 98*	*Approx Par*	*102 < P ≤ 110*	*P > 110*
Total observations	11,712	172	2,259	2,716	4,989	1,576
Panel A: Bid Prices						
Negative calls	5,630	172	2,185	1,921	1,270	82
	(48.1%)	(100.0%)	(96.7%)	(70.7%)	(25.5%)	(5.2%)
Negative puts	1,145	0	123	638	365	19
	(9.8%)	(0.0%)	(5.4%)	(24.5%)	(7.3%)	(1.2%)
Panel B: Spread-Adjusted Prices						
Negative calls	5,026	170	2,065	1,757	982	52
	(42.9%)	(98.8%)	(91.4%)	(64.7%)	(19.7%)	(3.3%)
Negative puts	602	0	25	344	222	11
	(5.1%)	(0.0%)	(1.1%)	(12.7%)	(4.5%)	(0.7%)

Frequency of negative puts and calls in five bid-price ranges for callable bonds with coupon rates of 8.75% or less. Put and call values obtained with the use of the all-STRIPS approach presented in eq. (1) from daily prices on 22 callable Treasury bonds covering the period 1990–1994. Numbers in parentheses are the percentage of observations in the indicated price range with negative implied option values.

bonds with coupons of 8.75% or less (the cutoff identified by Carayannopoulos). The effect is to eliminate approximately 56% of the sample. The resulting subsample is naturally less concentrated in premium issues, but there are still about twice as many observations at prices above 102 than there are below 98.

As seen in Table V, the overall frequency of negative option values is much higher, because essentially all of the discount observations are retained. However, an examination of the individual price ranges reveals that these lower coupon issues imply put and call values with essentially the same frequencies as the entire sample. There is little or no evidence of a significant coupon effect.

ARE CALLABLE TREASURY BONDS MISPRICED?

Researchers who have examined option values implied by callable Treasury bond prices agree on one thing: Negative option values appear to exist in the data. The differences that exist concern the frequency and economic significance of the apparent negative values. In evaluating the evidence, it is useful to review some of the reasons negative values may exist.

First of all, there is noise in the data. The cash bond and STRIPS data are collected by different entities, they are not synchronous, and they are based on dealer quotations, not transactions. Furthermore, the

STRIPS data are supplied by a single dealer, whereas cash bond data are representative prices based on a survey of dealers. As a result, some percentage of negative values will likely be observed in situations where the true option value is small but positive. By way of comparison, quoted stock option prices commonly imply intrinsic value violations, even in very actively traded instruments, and researchers routinely just discard such cases.

Second, the all-STRIPS approach used here and in Carayannopoulos (1995) tends to undervalue bonds on average, particularly discount bonds, and extreme misvaluations occur on occasion. Regardless of whether this is due to a tax disadvantage or some other reason, this downward bias adds to the noise problem and will contribute to the appearance of negative option values, particularly negative call option values.

Third, many of the callable bonds examined here and in other studies are older issues and/or relatively small issues.[12] Small size and old age are widely viewed as proxies for illiquidity in the Treasury market (e.g., Sarig and Warga, 1989). There is a general tendency for prices on smaller, older, less liquid Treasury issues to be quoted at prices that imply a small premium. It may seem odd that an illiquid issue would command a higher price, but Beim (1992) attributes it to a scarcity effect. When a customer requests a quote on a scarce issue, a dealer must supply it without knowing whether the customer is buying or selling. If the dealer is on the sell side and does not have the issue in stock, he or she must borrow it, typically via a reverse repurchase agreement, to make delivery. As explained in Duffie (1996), there can be substantial costs associated with a reverse when the issue is scarce. Given this, the dealer would prefer to be on the buy side and thus rationally quotes higher bid and ask prices. Here again, the fact that these are quoted prices rather than transactions prices may play a role.

Thus, there are a variety of reasons why negative option values can appear to exist when, in fact, the true value is positive. On the other hand, episodes of significant overvaluation in the Treasury market have been documented in, e.g., Cornell and Shapiro (1989) and Jordan and Jordan (1996). It is certainly not impossible that at least some of the negative option values are truly indicative of misvaluation.

For example, in a completely different context, Jordan and Kuipers (1997) show that, during the period covered in this study, the three callable bonds maturing in 2009-14 (the last three ever issued) exhibit eco-

[12]Some of the callable bonds examined in this and other studies were issued in the early 1970s with less than $1 billion in face value. In contrast, the typical noncallable bond in these studies has an outstanding face value of $10 billion or more.

nomically significant negative put option values nearly every day for eight consecutive months as the direct result of the three becoming, by a wide margin, the cheapest-to-deliver issues on the CBT Treasury bond futures contract. In fact, of the 1,061 negative (spread-adjusted) put values in Table IV, almost 300 are due to this one episode. Thus, this is one reason that negative option values can occur, and other circumstances may lead to similar results.

Overall, the analysis here, when combined with that in previous studies, suggests that if callable Treasury bonds are systematically mispriced, then that mispricing occurs infrequently, it is confined to lower-priced issues, and it appears in the form of negative implied call option values. Both theoretical and empirical considerations in other articles and additional empirical results here suggest that these are the circumstances under which the mismeasurement problem is most severe, so the economic significance of these unusual prices is not clear. Perhaps future research will go beyond comparisons of static bid–ask spreads and focus on whether the negative call values actually represent economically significant trading opportunities, and, if the conclusion is that they do, attempt to provide an explanation.

CONCLUSION

This study examines the pricing of callable Treasury bonds over the period 1990–1994 to determine whether negative embedded option values are observed. Based on almost 300,000 comparisons of callable bond prices to estimated noncallable bond prices, negative option values large enough to overcome the bid–ask spread occur 8.9% of the time. Negative implied put values are observed only 4.0% of the time, but negative implied call values occur in 18.9% of the cases examined.

Observed negative call values are heavily concentrated in bonds selling near par or below. For issues selling at 98% of par or less, over 90% imply negative call option values. However, when the empirical method used to estimate implied option values, which relies entirely on U.S. Treasury STRIPS prices, is investigated, the results suggest that implied call options will be systematically undervalued by a small amount on average and that extreme misvaluations occur on occasion. This result, along with a variety of theoretical and empirical considerations in other studies, suggests that, for callable bonds selling at a discount, negative option values can appear to exist when the true value is small, but positive. Thus, the economic significance of the effect is unclear.

Based on a reconstruction, this article also shows that a previous study by Carayannopoulos (1995) apparently reverses implied puts and calls, and, consequently, misstates the relative frequency with which negative put and call values are observed. A coupon effect, in which negative implied option values are concentrated in issues with coupons of 8.75% or less, appears to exist as a result of this misclassification. The findings in this study strongly suggest instead that the coupon rate effect is simply a reflection of the premium/discount status of the callable bonds examined.

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