The Effect of Prepayment Modeling in Pricing Mortgage-Backed Securities

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Abstract

Mortgage-backed securities (MBS) are a capital market innovation that gained popular acceptance in the 1980s and are even stronger in the 1990s. These instruments have revolutionized real estate mortgage markets and are popular investments for both individual and institutional investors. However, the pricing of these securities is subject to uncertainty due to the existence of the mortgage prepayment option. This study describes the options-based model that can be used to price MBS and details possible prepayment functions that can be incorporated into the model. Four different prepayment functions are suggested because the nature of the prepayment function is critical to pricing MBS accurately. Modifications to existing models are proposed.

Introduction

An important capital market innovation gaining popular acceptance in the 1980s has been the mortgage-backed security and its derivatives. These securities have been the target of considerable analysis by both investment bankers and academics; however, their valuation remains an unresolved issue in large part because of the mortgage borrower's prepayment behavior.[1] Prepayment options have been modeled as call options (Brennan and Schwartz 1977), but many of the assumptions implicit in these option-pricing models are violated. An alternative method of handling prepayment options is to formulate a model that modifies cash flows in the valuation process. This paper, using the latter approach in an options-based framework, will apply different prepayment functions to the valuation of mortgage-backed (MBS) securities in order to evaluate important factors and the sensitivity of these factors in pricing MBS.

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[1]When discussing the pricing of mortgage-backed securities with major New York investment banking firms, we found these firms very cooperative except regarding their own prepayment models. These models are considered proprietary because each firm has individually and secretly developed its own model.

Mortgage-Backed Securities

MBS are created through a process of securitization in which mortgage originators sell mortgages to private firms or government agencies such as the Government National Mortgage Association (GNMA) or government-sponsored enterprises (GSE) such as Fannie Mae or Freddie Mac. These mortgages are packaged into relatively homogeneous pools and placed in the custody of a trustee. The pools are used as collateral for the issuance of mortgage-backed securities. MBS convey ownership of an individual interest in the mortgage pool. Investors receive their share of the monthly interest, principal, and prepayment of principal payments collected from mortgages in the pool. Investors are usually guaranteed full and timely payment of interest and principal by the government agency, GSE, or a private insurer.[2]

The effect on the market of GNMA, Fannie Mae, and Freddie Mac is to increase the amount of loanable funds available for housing by stimulating a secondary mortgage market. By providing guarantees that eliminate default risk to the investor, these government agencies or enterprises help loan originators convert individual mortgages into more liquid ssecurities.

There are many derivative products of MBS, each fulfilling a different investor need. These products include collateralized mortgage obligations (CMO); real estate mortgage investment conduits (REMIC); mortgage cash flow obligations (MCF); guaranteed mortgage certificates (GMC); and stripped mortgage-backed securities in which the interest-only (IO) and principal-only (PO) portions of the security are sold separately. The nature of these derivative products—in which individual cash-flow components are priced separately—magnifies the importance of correctly pricing MBS.

The valuation of MBS and their derivatives is very sensitive to the prepayment behavior of mortgages. The borrower's option to prepay a mortgage loan at any time makes it necessary to adapt performance measures that are not needed for the valuation of other fixed-income securities.

The Options-Based Model

Instrument prices with embedded or explicit options, such as the prepayment option included in mortgages or MBS and their derivative products,

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[2]GNMA as a government agency carries the full faith and credit of the U.S. government with its guarantee; however, Fannie Mae and Freddie Mac as government-sponsored enterprises carry only an implicit guarantee of the U.S. government. Thus, there is uncertainty about the U.S. government's backing of Fannie Mae and Freddie Mac.

are highly dependent on the expected level and pattern of future interest rates and cash flows. Thus, an analytical technique called an options-based model will be used to price these securities.

The options-based model utilizes the relationship between embedded options and an interest rate process. The interest rate process forecasts how interest rates are expected to change in the future and is a primary determinant of mortgage prepayments.

Because of the prepayment option, future mortgage cash flows are dependent on future interest rates as well as other factors. Thus, some form of interest rate forecast is necessary to determine future cash flows. Interest rates are notoriously difficult to predict, however, and relying on a single interest rate would be rather precarious. Therefore, the options-based model simulates many possible interest rate scenarios.

Each possible interest rate path has an associated cash flow scenario via the prepayment function, which translates interest rates into prepayment rates. Each path has a unique cash flow pattern given prepayments and the amortization schedule of the mortgage. Further, interest rates of each path are used to discount that path's associated cash flows to the present, resulting in a theoretical market price for each interest rate scenario. Assuming that each path is equiprobable, the theoretical price of the security is the average of the prices associated with various interest rate scenarios.[3] Thus, arriving at the theoretical price incorporates a wide variety of possible option scenarios and results in the exercise of the prepayment option. In this way, the model evaluates the prepayment option in a fair and explicit manner.

The U.S. Treasury Term Structure

The options-based model used in this paper has three different components. The first component uses the observable par value U.S. Treasury yield curve and an exponential spline similar to that used by Vasicek and Fong (1982) to develop a treasury term structure (the yield on zero coupon treasury securities) for 360 future months. The treasury term structure is the basis for estimating all future implied forward rates and for the discount rate associated with each future monthly cash flow from the mortgage-backed security.

[3]The options-based model is a simulation approach to pricing mortgages and mortgage-backed securities. As is typical of simulation approaches, each iteration or interest rate scenario is assumed to be equally probable.

The spline methodology develops a term structure that is a continuous function of time. Using this function, monthly implied forward rates are calculated 30 years into the future at monthly intervals:

$$ {}_z f_1 = \frac{(1 + {}_0r_{z+1})^{z+1}}{(1 + {}_0r_z)^z} - 1 \qquad z = 1, \ldots, 360 \tag{1}$$

where ${}_0r_z$ is the z-period estimated annualized term structure rate and, ${}_zf_1$ is the annualized one-month forward rate.

Fama (1984) and others have argued that calculated implied forward rates overestimate interest rate expectations since "liquidity/term premia" are embedded in the term structure. However, using the entire series of spot rates to estimate each successive one-month forward rate will result in very little upward bias because the increase in the term premia from one month to the next is very small.

The Interest Rate Process

The second critical component of the options-based model is the interest rate process. Interest rate paths influence the cash flows of mortgage-backed assets and their respective discount values, so the interest rate dynamics used in the model must closely conform to observed interest rate behavior.

Using the interest rate process developed by Luytjes (1990) at the Office of Thrift Supervision (OTS), the modeling effort attempts to describe interest rate movements through time. This model uses a broad class of interest rate processes commonly known as "constant elasticity of variance." The interest rate process, developed using U.S. Treasury data from 1980 through 1990, incorporates a mean reversion process that is modified by the shape of the Treasury term structure.

Most mean reversion processes exhibit a rather restrictive assumption that regardless of how far the rate drifts from its mean-reverting value, the rate will return at a given speed toward the mean. However, a term structure that is strongly upward or downward sloping would suggest upward or downward sloping pressure on the short rate rather than a strong pull back to the mean. This situation would suggest that the term structure may cause a short-run modification to the mean-reverting rate. The following model allows the level and slope of the term structure to influence short-rate dynamics:

$$ \ln r_t^* = \delta\,(\ln i_{t-1} - \ln \gamma) + (1 - \delta)\ln LM \tag{2}$$

where

r^*_t = the temporary "mean-reverting" rate at time t,
ℓn = the natural logarithm,
i_{t-1} = last month's long (five-year zero) treasury rate,
γ = the long-run equilibrium ratio of the long rate to the short rate,
LM = the long-run equilibrium rate to which r^*_t tends, and
δ = a parameter between zero and one allowing the level and slope of last month's yield curve to influence r^*_t. The speed of adjustment toward LM is controlled by $(1 - \delta)$.

The interest rate process used in the options-based model must generate both a one-month forward rate (r_t) and a long (five-year) forward rate. The long rate is used to drive the prepayment function. Thus, a description of long-rate dynamics is needed:

$$\ell n\, i_t = \theta\,(\ell n\, r_{t-1} - \ell n\, \gamma) + (1 - \theta)\ \ell n\, i_{t-1} + \alpha\, S_t + W_t \qquad (3)$$

where

r_t = the one-month treasury rate at time t,

θ = a parameter between zero and one that allows the long rate to move toward its equilibrium relationship with the short rate (γ),

$S_t = \sigma Z_t$,

σ = the standard deviation of changes in the short rate (r) used to scale Z,

Z_t = a random shock drawn at month t from a unit normal distribution,

α = a parameter between zero and one that introduces a correlation between short- and long-rate shocks, and

W_t = an independent mean-zero random shock that, when taken with α and S_t, determines the volatility of the long rate.

Equations (2) and (3) have specified the process for the temporary mean-reverting rate (r^*_t), and for the long-term, five-year, forward rate. The remaining task is to specify the process for the short-term, one-month, forward rate (r_t).

$$\ell n\, r_t = \beta\ \ell n\, r^*_t + (1 - \beta)\ \ell n\, r_{t-1} + S_t \qquad (4)$$

where

β = a parameter between zero and one governing the speed of adjustment to $r,^*$ and other variables are as previously defined.

The interest rate specification represented by equations (2), (3), and (4) has several appealing properties. First, it is more general than the process typically found in the literature. A number of simpler processes may be specified by letting certain parameters equal zero. Second, the short-rate dynamics integrate the modified expectations hypotheses with the mean-reverting character of short rates. This integration allows for more rate diffusion than would be exhibited by simple mean-reverting processes. In addition, the long-run normal slope of the yield curve, τ, can be estimated. This estimate corresponds to the liquidity or term premium that compensates longer term lenders for interest rate risk.

The data used in the estimation of the joint process described by equations (2), (3), and (4) were the one-month and five-year zero-coupon U.S. Treasury rates for January 1980 through December 1990. These two rates for each month during the period were obtained from the treasury yield curve and the spline methodology defined in the first part of the options-based model.

Equation (2) was substituted into equation (4), and the resulting nonlinear equation was estimated using nonlinear least squares. The residuals for the monthly data exhibited strong serial correlation. Thus, they were analyzed using Box-Jenkins diagnostics, and the result suggested second-order autocorrelation. As a result, the error term of equation (4) was specified as

$$S_t = \rho_1 S_{t-1} + \rho_2 S_{t-2} + \mu_t \tag{5}$$

wheret μ_1 is a mean-zero normally distributed random variable, 1 is the parameter governing the first-order serial correlation, and ρ_2 is the parameter governing the second-order serial correlation.

Given the results of the short-rate process, the long-rate equation (3) was tested and the residuals exhibited similar behavior. Consequently, the error term of equation (3) was specified as

$$W_t = \phi_1 W_{t-1} + \phi_2 W_{t+-2} + v_t \tag{6}$$

where v_t is a mean-zero normally distributed random variable, $_1$ is the parameter governing the first-order serial correlation, and $_2$ is the parameter governing the second-order serial correlation.

Given the complete specification of equations (2) through (6), the short-rate and long-rate equations were estimated over the 11-year, 132-month period using generalized differencing. See table 1 for the parameters chosen to represent the process.

Table 1. **Interest-Rate Process Parameters**

Parameter	Value
β	0.095
δ	0.750
γ	1.210
LM	0.073
ρ_1	0.600
ρ_2	–0.270
θ	0.060
φ_1	0.500
φ_2	–0.300

Cash Flow Component

The third component of the options-based model is the cash flow for each of the simulated interest rate paths. The cash flows received during any specific period from a mortgage asset depend on the interest rate on the remaining principal, the normal principal amortization, and prepayment of the loan principal. The uncertainty of cash flows results from the prepayment of principal. Thus, a major factor in pricing of mortgage-backed securities is the correct specification of the mortgage prepayment model.

Review of Prepayment Models

In the past, the price of MBS was computed by assuming complete prepayment in the 12th year.[4] There is now evidence that the prepayment function is more complex and is determined by a number of different factors. Borrowers prepay their mortgages for many social reasons. Individuals usually buy houses when starting a family and may sell them after the children have grown and left. They also sell when the family breaks up because of divorce or job transfers. Economic conditions have an influence. Booming regions attract people from depressed areas. These people either sell or abandon houses to move to more prosperous

[4]This convention is still used to arrive at Ginnie Mae yields.

areas. Because some regions are more sensitive to such changes, the prepayment rate varies in different geographical regions.[5]

The age of the pool also contributes to the pattern of prepayments. Up to a point, older pools display higher prepayments for many of the same social reasons that affect borrowers (Milonas and Lacey 1988). Even though borrowers do not plan to sell when they first buy a home, prepayments are predominantly higher during the first few years of the pool. As the pool ages, the social reasons for prepayment decline.

Borrowers also prepay to take advantage of a drop in interest rates, and this event is a major factor in predicting prepayment behavior. A mortgage can be viewed as an annuity of fixed payments over a definite time period with the option of prepayment by the borrower. In a static model, if the market rate of interest is equal to the coupon rate of the mortgage, the principal balance of the contract is equal to the present value of future payments. When the market interest rate is different from the coupon rate, the present value of future payments must be discounted at the market rate. If the market rate is higher than the coupon rate, there is no reason in a static model, other than social factors, to refinance, because the present value of future payments will be higher than the balance of the contract. Only when the market or refinancing rate is lower than the coupon rate is it advantageous for the borrower to refinance.

Refinancing is not a costless activity, and the cost can vary among borrowers and time frames.[6] The cost of appraisals, credit checks, and certain loan fees must be considered. Mortgages may include a penalty for prepayment and a “no assumption” clause. The borrower must analyze these costs to determine whether prepayment is economically advisable. The decision to refinance should be made if the refinancing costs plus the present value of the new payments discounted at the new interest rate are less than the present value of payments at the existing rate.

Follian and Tzang (1988) used such a model to determine the interest rate differential needed to justify refinancing. They found that a spread of 60 basis points is needed for a borrower to refinance with an average

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[5] Milonas (1987) stated that California has a high divorce rate; Florida and California have booming economic environments; and industrialized regions such as Michigan, or states that are heavily dependent on oil production, such as Texas, Louisiana, and Oklahoma, are economically unstable.

[6] Richard and Roll (1989) found evidence that these refinancing costs also vary over time. After interest rates have stopped declining and start to rise, some prepayment occurs indicating that refinancing costs have lowered to the point where borrowers should refinance.

holding period of 10 years. They also noted that this differential declines as the expected holding period increases. One problem with this method is that it ignores the fact that interest rates could continue to decline. If borrowers knew that interest rates would keep dropping, they could save more money by waiting to refinance until rates cease to fall. This possibility obviously is an opportunity cost to the borrower who refinances.[7]

The borrower's decision to wait and the time needed by financial institutions to process a refinancing request create a lag. If interest rates drop significantly and many borrowers try to refinance, the onslaught may cause a backlog at the financial institution. Therefore, the recording date of the refinancing may be much later than the original date of the borrower's decision.[8]

Option-pricing theory has been the basis for many studies into the prepayment behavior of borrowers. But often the assumptions underlying the theory are violated. Most option-pricing models assume that the option holder acts only to maximize the value of the option. However, borrowers may refinance for other reasons, such as the social reasons discussed above, or they may sell at what they consider the peak of a booming housing market in order to maximize their total wealth.

In an attempt to value prepayment options, MBS have been compared to callable bonds. Brennan and Schwartz (1977) developed a methodology for pricing savings bonds, retractable bonds, and callable bonds. Subsequently, Buser, Hendershott, and Sanders (1987) extended this methodology to include the borrower's option to prepay a 30-year mortgage. Follian, Scott, and Yang (1988) reinforced the idea that a borrower's option of prepaying a mortgage is similar to a call option and that this call option has a positive value.

The term structure of interest rates is a determining factor in a borrower's decision to refinance. If current interest rates are higher than the coupon rate, the borrower will not opt to refinance. If current rates are lower than the coupon rate, the decision to refinance could depend on how long rates have been down and on whether they are expected to drop still further. As with other option-pricing models, the volatility of interest rates greatly influences the value of the option. If interest rates do not fluctuate, there is no volatility and the value of the option is zero. With MBS, there is economic value in prepaying only if interest rates go down.

[7] For this reason, it is apparent that a static analysis of the prepayment option will underestimate the value of the option. Higher interest rate volatilities will increase the value of the option.

[8] This factor, influencing the value of MBS, was also examined by Milonas and Lacey (1988).

Recent Prepayment Models

Recently, there has been a proliferation of prepayment models in the financial literature. These models include those by Asay, Guillaume, and Mattu (1987); Brazil (1988); Carron and Hogan (1988); Chinloy (1989, 1991); Davidson, Herskovitz, and Van Drunen (1988); Giliberto and Thibodeau (1989); Lacey and Milonas (1989); Richard and Roll (1989); and Schwartz and Torous (1989). In order to evaluate a prepayment model's effect on the cash flows of MBS and their valuation, our options-based model will use the models developed by Asay, Guillaume, and Mattu (1987); Chinloy (1991); Schwartz and Torous (1989); and a Goldman Sachs model developed by Richard and Roll (1989) and modified by the Office of Thrift Supervision.[9] The only difference in the four models is the pattern of cash flows generated by each prepayment option.

The Asay, Guillaume, and Mattu Model

The model formulated by Asay, Guillaume, and Mattu (1987) incorporates only the spread between the prevailing market rate and the loan coupon rate, as

$$CPR = .3 - .16ATN\,[123.11\,(SPREAD +. 02)] \qquad (7)$$

where CPR is the conditional prepayment rate, ATN is the arctangent, and $SPREAD$ is the prevailing market loan rate less the loan coupon rate.

The Chinloy Model

Chinloy (1991) found three factors that were significant in explaining the monthly prepayment rate for GNMA mortgage-backed securities from January 1986 through May 1989. These factors are the average market rate on newly originated, fixed-rate mortgages (r), the contract rate (a), and the seasoning or age of the loan (t). Based on a Tobit specification, the results (with standard errors in parentheses) are

$$\begin{aligned} CPR = {}& 0.0813 - 1.7951\ (0.7635)r\ + 0.9063 \\ & (0.0688)\ \alpha + 0.0012\ (0.0024)t \end{aligned} \qquad (8)$$

Chinloy observed that age and seasoning do not affect the probability of prepayment.

[9]The parameters of the Goldman Sachs model were not reported in the Richard and Roll paper; however, they were estimated using prepayment output from Wall Street models.

The Schwartz and Torous Model

Like Green and Shoven (1986), Schwartz and Torous use a proportional-hazard model to estimate the influence of various explanatory variables on Ginnie Mae 30-year, single-family pool prepayment rates during the period of January 1978 to November 1987. Unlike Green and Shoven, Schwartz and Torous show the effects of seasoning and investigate the impact of interest-cost savings from refinancing. They also consider lagged refinancing rates, heterogeneity in mortgages, and seasonality. The maximum-likelihood estimates of the prepayment function (with jackknifed standard deviations in parentheses) are

$$CPR = \{\gamma p(\gamma t)^{p-1}/[1 + (\gamma t)^{p}]\} \exp\left(\textstyle\sum_{h=1}^{4} \beta_h V_h\right) \tag{9}$$

where

$\gamma = 0.01496\ (0.00110)$,

$p = 2.31217\ (0.13919)$,

$\beta_1 = 0.38089\ (0.06440)$,

$\beta_2 = 0.00333\ (0.00134)$,

$\beta_3 = 3.57673\ (0.34504)$, and

$\beta_4 = 0.26570\ (0.32870)$.

$$V_1(t) = c - 1\,(t - s),\ s \geq 0 \tag{10}$$

introduces the effect of refinancing costs on the mortgagor's prepayment decision. The mortgage contract rate is c;   is the long-term treasury rate for month t with an s-month lag. A lag of three months ($s = 3$) was used in the model.

$$V_2(t) = [c - 1\,(t - s)]^3,\ s \geq 0 \tag{11}$$

introduces the possibility that prepayments may further accelerate when refinancing rates are sufficiently lower than the mortgage contract rate.

$$V_3(t) = \ln\,(AO_t / AO_t^*) \tag{12}$$

determines the proportion of a GNMA pool previously prepaid. The dollar amount of the pool outstanding at time t is AO_t. AO^*t is the pool's principal that would prevail at t in the absence of prepayments but would reflect the amortization of the underlying mortgages.

Since more residential real estate transactions occur in spring and summer than in the fall and winter, seasonality influences prepayments:

$$V(t) = \begin{cases} +1 & \text{if } t = \text{May through August} \\ 0 & \text{if } t = \text{September through April.} \end{cases} \tag{13}$$

The Modified Goldman Sachs Model

The Goldman Sachs model developed by Richard and Roll and modified by the OTS, captures four important economic effects.[10] These effects are (1) the refinancing incentive; (2) seasoning or age of the mortgage; (3) the month of the year (seasonality); and perhaps the least understood, (4) the pool burnout effect.

These four factors are combined in a multiplicative function as

$$\begin{aligned} CPR = {} & \text{(Refinancing Incentive)(Seasoning Factor)} \\ & \text{(Month Factor)(Pool Burnout Factor)} \end{aligned} \tag{14}$$

where CPR is the conditional annual prepayment rate.

This model measures the refinancing incentive as the weighted average of the mortgage coupon rate divided by the mortgage refinancing rate. In particular, the model used is[11]

$$\begin{aligned} RI = {} & .31234 - .20252{*}ATN\,(8.157{*}[-(C + S) / (P + F) + \\ & 1.20761]) \end{aligned} \tag{15}$$

where RI is the conditional prepayment resulting from the refinancing incentive, ATN is the arctangent function, C is the MBS average coupon rate, S is the loan-servicing rate taken out of the pool, P is the refinancing rate, and F is the additional refinancing cost associated with refinancing the mortgage.

A weighted average of recent values of C and P is used to capture the mortgage-lending delays homeowners face when responding to refinancing incentives. This lag is approximately three months.

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[10] See Office of Thrift Supervision (1989) for a similar description. This model was also used by Spahr, Luytjes, and Kale (1991).

[11] The parameters of the refinancing incentive, the design of the seasoning factor, the month factor, and the pool burnout factor were estimated using prepayment output from Wall Street models.

Mortgage prepayment rates are very low for newly created mortgages but rise steadily as mortgages age. This increase may be the result of two factors: (1) new mortgages are not likely to be prepaid for a number of months because, in most cases, homeowners will not relocate immediately; and (2) new buyers are unlikely to refinance immediately because of transaction costs and inconvenience. Thus, the modified Goldman Sachs model assumes that the seasoning factor will begin at zero at month zero and linearly approach one over a specified seasoning period. A loan was found to be fully seasoned after 30 months.

Prepayments generally peak in the later summer months and trough in the winter because household moves follow this seasonal pattern. The model incorporates a sine wave:

$$Month\,factor = 1 + 0.2 * Sin\,\{(\pi/2 * [m + t - 3)/3 - 1]\} \tag{16}$$

where m is the month (1,2,—,12) in which the MBS are priced.

Pool burnout results because not all mortgagors in a given pool prepay identically. Some mortgagors will prepay as soon as refinancing rates are somewhat lower than their mortgage coupon; others will require refinancing rates to drop even further; and still others will never prepay. Thus, as a mortgage ages and there have been sufficient opportunities to prepay (i.e., the option to prepay has been deep in the money), those who are quick prepayers will be "wrung out," leaving slower payers in the pool. The burnout function depends on the entire history of the pool. The more often the option has been in the money, the more rapidly pool burnout will occur:

$$Pool\ burnout\ factor = \exp(-0.115B) \tag{17}$$

where B is a function of the ratio of the mortgage coupon rate to the refinancing rate.

In the modified Goldman Sachs model, pool burnout does not begin until after the seasoning factor reaches one. In an environment of fluctuating interest rates where rates rise and fall above and below the loan's coupon rate, pool burnout would occur most rapidly when rates are significantly below the coupon rate. Thus, it is assumed that prepayment rates decline exponentially when interest rates are less than the mortgage coupon rate.

Options-Based Pricing Results

Thirty-year mortgage-backed securities and their interest-only (IO) and principal-only (PO) strips were priced using the options-based model for five different prepayment functions and for coupon rates ranging from 7 percent to 15 percent. The five prepayment functions are for the Asay, Guillaume, and Mattu model; the Chinloy model; the Schwartz and Torous model; and the modified Goldman Sachs model, both with and without the burnout component. The yield curve was used in pricing 30-year residential mortgage pools backing MBS is shown in table 2.

Table 2. **Yield Curve for Pricing 30-year Residential Mortgage Pools**

Maturity	Yield on Par U.S. Treasuries (percent)
1 month	5.47
3 months	5.52
6 months	5.80
1 year	6.11
2 years	6.68
3 years	7.10
4 years	7.30
5 years	7.68
7 years	7.93
10 years	8.09
20 years	8.28
30 years	8.31

Given this yield curve and the observation that a 30-year mortgage-backed security with 9.10 percent coupon was priced at par, it was determined using the modified Goldman Sachs model that the options-adjusted spread (OAS), was 1.17 percent.

This study assumes that all different coupon MBS are priced using an OAS of 117 basis points.[12] This assumption allows the price to vary according to the specific prepayment function in the options-based model and compares each prepayment function with the modified Goldman Sachs model. An alternative methodology would be to price actual mortgage-backed securities found in the market and determine their individual OAS. This procedure would give each coupon mortgage-backed security a unique OAS. Each prepayment function would also have a unique OAS when compared with other prepayment models. Whether different coupon MBS have different OASs is an empirical question that cannot be adequately addressed unless the prepayment function is correctly specified. Thus, the objective of this paper is to demonstrate that prepayment functions suggested in recent literature

[12] Empirically, the OAS usually depends on the coupon; however, for the purpose of this paper, 117 basis points were used for all MBS. This assumption has no impact on the purpose of the paper, which is to demonstrate that the prepayment function is critical in pricing MBS.

may result in significantly different prices for different coupon MBS and their derivatives and that a correctly specified prepayment function may not yet exist.

See table 3 for a presentation of the prices for different coupon, 30-year mortgage-backed securities for the five different prepayment functions.[13]

Interest-only derivatives as a percentage of par value are priced lower in the Asay, Guillaume, and Mattu model than in any of the other four. The pattern of IO prices is highly dependent on the prepayment model. Three models (Asay and colleagues, Chinloy, and modified Goldman Sachs without burnout) suggest that IO prices increase up to a coupon rate of 9 1/2 percent to 10 percent and decrease for higher coupon rates. Conversely, the modified Goldman Sachs with burnout and the Schwartz and Torous models suggest that IO prices increase with increasing coupon rates. Since IO will be priced higher with slower prepayment rates, the modified Goldman Sachs with burnout and the Schwartz and Torous models both result in a substantial slowdown of prepayments as mortgages age. The burnout factor found in both of these models substantially affects the prices for IO.

Conversely, PO derivatives as a percentage of par value are priced higher for the Asay and colleagues' model, although in all models except for Schwartz and Torous, the price of POs increase for higher coupon rates. Since PO prices are higher the faster mortgages are prepaid, higher coupon mortgages will result in higher PO prices because of the increased prepayment incentive. Only the Schwartz and Torous model resulted in a reduction of PO prices with higher coupon rates. This model was very insensitive to prepayments caused by refinancing and was much more sensitive to pool burnout. Clearly, the Schwartz and Torous model, when used in the options-based framework, results in the most severe change in price for IO and the MBS with very little change in price for PO. These prices are not consistent with the premiums at which mortgaged-backed securities normally trade.

The modified Goldman Sachs and the Schwartz and Torous models will be compared by evaluating the technique and sensitivity with which each of the four factors (refinancing incentive, seasoning, seasonality, and burnout) is incorporated.

The refinancing incentive used by Schwartz and Torous includes a linear and cubic difference between the mortgage contract rate and the lagged

[13] In table 3, it is assumed that the market values of IO and PO sum to the market value of the MBS. In practice, we usually observe the two components having a higher value than the MBS to justify stripping the security.

Table 3. **Prices of IO, PO, and MBS with 55 Basis Points Mortgage Servicing, 360 Months to Maturity, and Different Prepayment Functions**

(percent)	IO	PO	Price
	Asay, Guillaume, and Mattu Model		
7.0	39.006	49.955	88.961
7.5	41.193	50.706	91.899
8.0	42.982	51.832	94.814
8.5	44.250	53.400	97.650
9.0	44.877	55.461	100.338
9.5	44.772	58.025	102.797
10.0	43.929	61.019	104.948
10.5	42.475	64.266	106.741
11.0	40.642	67.535	108.177
11.5	38.695	70.611	109.306
12.0	36.852	73.350	110.202
12.5	35.241	75.700	110.941
13.0	33.900	77.678	111.578
14.0	31.984	80.706	112.690
15.0	30.973	82.771	113.744
	Goldman Sachs Model Without Burnout		
7.0	49.206	36.151	85.357
7.5	51.800	37.326	89.126
8.0	53.996	38.830	92.826
8.5	55.739	40.667	96.406
9.0	57.000	42.809	99.809
9.5	57.789	45.203	102.992
10.0	58.144	47.769	105.913
10.5	58.143	50.416	108.559
11.0	57.880	53.050	110.930
11.5	57.449	55.594	113.043
12.0	56.924	58.002	114.926
12.5	56.336	60.242	116.608
13.0	55.823	62.300	118.123
14.0	54.933	65.843	120.775
15.0	54.459	68.648	123.107
	Goldman Sachs Model With Burnout		
7.0	52.953	31.004	83.957
7.5	56.341	31.503	87.844
8.0	59.452	32.270	91.722
8.5	62.230	33.320	95.550
9.0	64.637	34.645	99.282
9.5	66.663	36.212	102.875
10.0	68.323	37.970	106.293
10.5	69.662	39.852	109.514
11.0	70.749	41.781	112.530
11.5	71.661	43.688	115.349
11.5	71.661	43.688	115.349
12.0	72.458	45.527	117.985
12.5	73.191	47.269	120.460
13.0	73.900	48.894	122.794
14.0	75.393	51.748	127.141
15.0	77.177	54.037	131.214

Table 3. **Prices of IO, PO, and MBS with 55 Basis Points Mortgage Servicing, 360 Months to Maturity, and Different Prepayment Functions** *(continued)*

(percent)	IO	PO	Price
	Chinloy Model		
7.0	54.786	28.289	83.165
7.5	57.473	29.982	87.455
8.0	59.521	32.090	91.611
8.5	60.955	34.609	95.564
9.0	61.751	37.539	99.290
9.5	61.899	40.816	102.715
10.0	61.443	44.317	105.760
10.5	60.413	47.986	108.399
11.0	58.895	51.747	110.642
11.5	56.972	55.516	112.488
12.0	54.758	59.187	113.945
12.5	52.332	62.715	115.047
13.0	49.759	66.078	115.837
14.0	44.517	72.133	116.650
15.0	39.478	77.229	116.707
	Schwartz and Torous Model		
7.0	42.812	44.885	87.697
7.5	46.148	44.531	90.679
8.0	49.506	44.195	93.701
8.5	52.883	43.877	96.760
9.0	56.277	43.576	99.583
9.5	59.685	43.292	102.977
10.0	63.106	43.025	106.131
10.5	66.538	42.773	109.311
11.0	69.978	42.536	112.514
11.5	73.427	42.313	115.740
12.0	76.881	42.105	118.986
12.5	80.339	41.909	122.248
13.0	83.801	41.726	125.527
14.0	90.731	41.394	132.125
15.0	97.664	41.105	138.769

long-term treasury rate. The modified Goldman Sachs model incorporates the arctangent in its refinancing incentive. Given that historical prepayment rates have followed the arctangent shape as a function of the coupon rate and the refinancing rate, and the simplistic multiplicative approach of Goldman Sachs, this model is intuitively preferred to Schwartz and Torous.

The seasonality or month factor in the modified Goldman Sachs model uses a sine wave rather than the binomial function used by Schwartz and

Torous. Both approaches could be simplified with the multiplicative approach of modified Goldman Sachs if a simple normalized factor were used. For months with prepayments greater than normal, a factor greater than one would be used, and for months where prepayments were lower than normal, a factor less than one could be used. This approach would not require prepayments to follow a sine function or a zero-one designation in which each month of the year would include its own factor.

The pool burnout effect used by Schwartz and Torous is the ratio of the actual dollar amount of the pool outstanding to the dollar amount of the pool outstanding in the absence of prepayment. This approach is preferable to others. The ratio is the best proxy for measuring pool burnout short of tracing the history of refinancing rates from the time of the pool's creation.

It is our opinion that incorporating the above modifications into a prepayment model may more closely approximate actual prepayment behavior.

Conclusions and Recommendations

This paper has addressed the importance of the prepayment option found in essentially all mortgage-backed securities and their interest-only and principal-only derivatives. Each of the five prepayment models proposed in recent literature prices the mortgage-backed securities and its derivatives differently, and the prepayment model used is critical in correctly pricing these securities.

Each of the five models was applied to an options-based pricing framework that incorporates the current U.S. Treasury term structure, a constant elasticity of variance in the interest rate process, and prepayments. Two of the models—modified Goldman Sachs and Schwartz and Torous—assume that monthly prepayment rates depend on four effects—the refinancing incentive, the seasoning of the mortgage, seasonality, and the burnout effect. The other models included only some of these factors. The Asay, Guillaume, and Mattu model uses only the refinancing incentive.

Clearly, because of the wide range of values for IO, PO, and MBS, there remains considerable room for further research on prepayment rates, mortgages, and mortgage pools.

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